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with Credit Shocks.*

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# Money Velocity in an Endogenous Growth Business Cycle with Credit Shocks \*

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## Abstract

The explanation of velocity in neoclassical monetary business cycle models relies on a goods productivity shocks to mimic the data's procyclic velocity feature; money shocks are not important; and the financial sector plays no role. This paper sets the model within endogenous growth, adds exchange credit shocks, and finds that money and credit shocks explain much of the velocity variation. The role of the shocks varies across sub-periods in an intuitive fashion. Endogenous growth is key to the construction of the money and credit shocks since these have similar effects on velocity, but opposite effects upon growth. The model matches the data's average velocity and simulates most of the velocity volatility that is found in the data. Its underlying money demand is Cagan-like in its interest elasticity, so that money and credit shocks cause greater velocity variation the higher is the nominal interest rate.

**Keywords:** Velocity, business cycle, credit shocks, endogenous growth.

**JEL:** E13, E32, E44

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# 1 Introduction

Explaining velocity at business cycle frequencies involves a rich literature. Freeman and Kydland (2000), Hodrick et al (1991) and Cooley and Hansen (1995) endogenize money velocity in models with shocks to the goods sector productivity and the money supply. Cooley and Hansen call the procyclic behavior of US velocity "one of the most compelling features of aggregate data"(p.179). Their model reproduces this but its correlation of velocity with output is high compared to data. Here the goods sector productivity shock drives velocity changes, in a way similar to Friedman and Schwartz's (1963) velocity theory as based on the application of the permanent income hypothesis to money demand (p.44). A positive temporary output shock (productivity) causes income to rise temporarily while money demand depends on consumption demand and is not much affected by the temporary income increase; a procyclic velocity results. However the most common explanation of velocity, that it depends on monetary-induced inflation effects on the nominal interest rate, as in McGrattan (1998), has no role in explaining velocity at business cycle frequencies, as Wang and Shi (2006) note in their alternative search-theoretic approach to velocity. Also missing is a role for financial sector shocks (King and Plosser 1984), financial innovation (Ireland 1991), technological progress (Berger 2003), or deregulation (Stiroh and Strahan 2003).

The paper explains 75% of the variability of velocity seen in 1972-2003 US quarterly data, by confronting the problems of velocity movements that are too procyclic, that are little affected by money shocks, and that have no role for financial sector shocks. In particular, it adds shocks to the productivity of providing exchange credit, which is introduced instead of the trips-to-the-bank approach of Freeman and Kydland (2000) or the cash-good, credit-good framework in Hodrick et al (2006) and Cooley and Hansen (1995), and uses an endogenous growth framework instead of an exogenous growth one (Section 2). Money and credit shocks both positively affect velocity but affect growth in opposite ways (Section 3). This allows both shocks to get picked up by the shock construction process (Appendix), thereby inducing

a large role for the shocks in the velocity variation and a subsequently less procyclic velocity as the goods productivity shock is relatively less important. The velocity variance decomposition for post-1972 data show all three shocks playing large roles that vary by subperiod. Money shocks have the largest effect during the high inflation period of 1972-1982, as might be expected; credit shocks are relatively more important during the financial deregulatory period of 1983-1995, also as expected (Section 4). The results are discussed relative to other velocity studies (Section 5), with conclusions (Section 6).

## 2 Endogenous Growth with Credit

The representative agent economy is an endogenous growth extension of Benk, Gillman, and Kejak (2005), with a Lucas (1988) human capital investment technology causing growth. The agent allocates resources amongst three sectors: goods production, human capital investment, and exchange credit production as a means to avoid the inflation tax. There are three random shocks at the beginning of the period, observed by the consumer before the decision process, which follow a vector first-order autoregressive process for goods sector productivity,  $z_t$ , the money supply growth rate,  $u_t$ , and credit sector productivity,  $v_t$ :

$$Z_t = \Phi_Z Z_{t-1} + \varepsilon_{Zt} \quad (1)$$

where the shocks are  $Z_t = [z_t \ u_t \ v_t]'$ , the autocorrelation matrix is  $\Phi_Z = \text{diag}\{\varphi_z, \varphi_u, \varphi_v\}$  and  $\varphi_z, \varphi_u, \varphi_v \in (0, 1)$  are autocorrelation parameters, and the shock innovations are  $\varepsilon_{Zt} = [\varepsilon_{zt} \ \varepsilon_{ut} \ \varepsilon_{vt}]' \sim N(\mathbf{0}, \Sigma)$ . The general structure of the second-order moments is assumed to be given by the variance-covariance matrix  $\Sigma$ . These shocks affect the economy as described below.

The representative agent's period  $t$  utility over consumption  $c_t$  and leisure  $x_t$  is  $\frac{(c_t x_t^\Psi)^{1-\theta}}{1-\theta}$ , with  $\theta \geq 0$  and  $\Psi > 0$ . Output of goods ( $y_t$ ) is produced with physical capital ( $k_t$ ) that depreciates at the rate  $\delta_k$  and with effective labor, through Cobb-Douglas production functions. Investment ( $i_t$ ) is given by the accumulation equation  $k_{t+1} = (1-\delta_k)k_t + i_t$ . A unit of time is divided amongst leisure ( $x_t$ ) and work in goods production ( $l_t$ ), human capital investment ( $n_t$ ),

and exchange credit production ( $f_t$ ):

$$1 = x_t + l_t + n_t + f_t. \quad (2)$$

With  $h_t$  denoting human capital, the effective labor employed across sectors is  $l_t h_t$ ,  $n_t h_t$ , and  $f_t h_t$  respectively. Given  $A_H > 0$ ,  $\delta_h \geq 0$ , human capital accumulates with a labor-only technology (Lucas 1988):

$$h_{t+1} = (1 - \delta_h)h_t + A_H n_t h_t. \quad (3)$$

Let  $a_t \in (0, 1]$  denote the fraction of consumption goods that are purchased with money ( $M_t$ ); then the exchange constraint can be expressed as

$$M_t + T_t \geq a_t P_t c_t, \quad (4)$$

where  $M_t$  is the money stock carried from the previous period and  $T_t$  is the nominal lump-sum money transfer received from the government at the beginning of the current period. Exchange credit ( $q_t$ ) is produced by the consumer acting in part as a bank to provide a means to pay for the rest of the purchases, without having to hold cash in advance of trading, and instead paying off the debt at the end of the period; this gives that

$$q_t = c_t (1 - a_t). \quad (5)$$

The consumer deposits all income that is not invested, of  $y_t - i_t = c_t$ , in its bank, makes purchases of goods  $c_t$  with the cash and credit taken out of deposits  $d_t$ , where  $d_t = [(M_t + T_t) / P_t] + q_t = a_t c_t + (1 - a_t) c_t = c_t$ . As a bank, the consumer uses a case of the now-standard Clark (1984) financial services technology to produce the exchange credit  $q_t$ . Clark assumes a constant returns to scale function in labor, physical capital, and financial capital that equals deposited funds.<sup>1</sup> Here for simplicity no physical capital enters; with  $A_F > 0$  and  $\gamma \in (0, 1)$ , the CRS production technology is  $q_t = A_F e^{v_t} (f_t h_t)^\gamma d_t^{1-\gamma}$ , where  $v_t$  is the shock to factor productivity; since deposits equal consumption, this can be written as

$$q_t = A_F e^{v_t} (f_t h_t)^\gamma c_t^{1-\gamma}. \quad (6)$$

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<sup>1</sup>Many studies have empirically verified this CRS specification including deposits as the third factor, and this specification has become dominant in current work, for example Wheelock and Wilson (2006).

Solving for  $q_t/c_t$  from equation (6), substituting this into the relation  $a_t = 1 - (q_t/c_t)$  from equation (5), and substituting this relation for  $a_t$  back into the exchange constraint (4), yields an exchange constraint analogous to a shopping time constraint as extended to endogenous growth:<sup>2</sup>

$$M_t + T_t \geq [1 - A_F e^{v_t} (f_t h_t / c_t)^\gamma] P_t c_t. \quad (7)$$

Let  $w_t$  and  $r_t$  denote competitive wage and rental rates. Nominal wages ( $P_t w_t l_t h_t$ ) and rents ( $P_t r_t k_t$ ) plus any unspent cash ( $M_t + T_t - a_t P_t c_t$ ), make up the consumer's income, while set-aside cash ( $M_{t+1}$ ) plus end-of-period credit debt payments [ $c_t (1 - a_t)$ ], and investment ( $i_t$ ) are expenditures:

$$P_t w_t l_t h_t + P_t r_t k_t + T_t + M_t - M_{t+1} - P_t c_t - P_t k_{t+1} + P_t (1 - \delta_k) k_t \geq 0. \quad (8)$$

The government transfers a random amount  $T_t$  given by

$$\frac{T_t}{M_t} = \Theta_t = \Theta^* + e^{u_t} - 1 = \frac{M_{t+1}}{M_t} - 1, \quad (9)$$

so that  $\Theta^*$  is the stationary gross growth rate of money.

The competitive firm maximizes profit given by  $y_t - w_t l_t h_t - r_t k_t$ , with production technology  $y_t = A_G e^{z_t} k_t^{1-\alpha} (l_t h_t)^\alpha$ . Then

$$w_t = \alpha A_G e^{z_t} \left( \frac{k_t}{l_t h_t} \right)^{1-\alpha}; \quad (10)$$

$$r_t = (1 - \alpha) A_G e^{z_t} \left( \frac{k_t}{l_t h_t} \right)^{-\alpha}. \quad (11)$$

**Definition of Equilibrium** Denoting the state of the economy by  $s = (k, h, M, z, u, v)$ , and with  $\beta \in (0, 1)$ , the representative agent's optimization problem can be written in a recursive form as:

$$V(s) = \max_{c, x, l, n, f, k', h', M'} \left\{ \frac{(cx^\Psi)^{1-\theta}}{1-\theta} + \beta EV(s') \right\} \quad (12)$$

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<sup>2</sup>Solve  $f_t h = g(c_t, M_{t+1}/P_t)$ . Then the main shopping time restrictions follow: that  $g_1 \geq 0$  and  $g_2 \leq 0$ , as shown in Gillman and Yerokhin (2005); but here the specification of  $f_t h$  results from the credit technology rather than a pre-determined interest elasticity of money demand as in shopping time models.

subject to the conditions (2), (3), (7) and (8). Define the competitive equilibrium as a set of policy functions  $c(s)$ ,  $x(s)$ ,  $l(s)$ ,  $n(s)$ ,  $f(s)$ ,  $k'(s)$ ,  $h'(s)$ ,  $M'(s)$ , pricing functions  $P(s)$ ,  $w(s)$ ,  $r(s)$  and the value function  $V(s)$ , such that (i) households maximize utility  $V(s)$ , given the pricing functions and that the policy function  $V(s)$  solves the functional equation (12); (ii) firms maximize profits, with the functions  $w$  and  $r$  given by (10) and (11); (iii) the goods and money markets clear, in equations (8) and (9).

**Description of Equilibrium** Here the focus is on the effects of shocks on velocity, the output growth rate, and the capital to effective labor ratio across sectors. Equilibrium money demand, and its velocity, is solved primarily from the first-order condition with respect to the choice of hours employed in credit production, this being the additional condition compared to a cash-only economy. Combined with equations (4) to (7), and other conditions to determine the constraint multipliers, the consumption-normalized money demand is given by

$$\frac{M_{t+1}}{P_t c_t} = a_t = 1 - (A_F e^{v_t})^{1/(1-\gamma)} \left( \frac{\gamma R_t}{w_t} \right)^{\gamma/(1-\gamma)}. \quad (13)$$

A positive money supply growth rate shock increases  $R_t$  through its inflation rate component and lowers normalized money demand (raises consumption velocity). A positive credit productivity shock  $v_t$  reduces money demand directly (raises consumption velocity). A positive goods productivity shock increases  $w_t$  and  $R_t$  through equations (10) and (11), and the Fisher equation of interest rates, by which the real interest rate  $r_t$  affects the nominal interest rate  $R_t$ ; the net effect on  $R_t/w_t$  is small since there is no effect of this shock on  $r_t/w_t$ .

The interest elasticity magnitude (denoted  $\eta$ , where  $w_t$  is held constant) is  $\eta = [\gamma/(1-\gamma)](1-a_t)/a_t$ ; this rises with  $R_t$  as in the Cagan (1956) model;  $\partial\eta/\partial R = \frac{\eta\gamma}{a_t R_t(1-\gamma)} > 0$ . With the baseline calibration values of  $a_t = 0.224$ , and  $\gamma = 0.13$ , then at  $R = 0.10$ , the interest elasticity is  $-0.52$ . The importance of the elasticity can be seen by considering that there is a bigger increase in velocity from an interest rate increase, the higher is the interest rate (and elasticity);  $\partial^2(1/a_t)/\partial R_t^2 = \frac{\eta}{(a_t R_t)^2} \frac{2\gamma-a_t}{1-\gamma} > 0$  for  $a_t < 2\gamma = 0.26$ ,

and  $w_t$  constant. And also a credit shock causes a bigger change in velocity the higher is the interest rate (and elasticity); with  $w_t$  and  $R_t$  constant,  $\partial(1/a_t)/\partial v_t = \frac{\eta}{\gamma a_t} > 0$  for  $R_t > 0$ ; and with  $w_t$  constant,  $\partial^2(1/a_t)/(\partial R_t \partial v_t) > 0$  for  $R_t > 0$ . This can explain, for example, why there would be a large response to the model's velocity from deregulation in the early 1980s when interest rates were higher: nominal interest rates fell rapidly after 1981 but velocity stayed high as deregulation began.

Note that in Cooley and Hansen (1995), the comparable normalized money demand is equal to  $\phi/[1 + R_t(1 - \phi)]$ , where  $\phi$  is a preference parameter for cash goods. A positive money supply shock and goods productivity shock both increase  $R_t$  and reduce the money demand; but with their calibrated value of  $\phi = 0.84$ , and say  $R_t = 0.10$ , the interest elasticity of the normalized money demand is  $-0.016$ , compared to  $-0.52$  in our model.

The total effect on income velocity depends not only on  $\frac{M_{t+1}}{P_t c_t}$  but also on the income-consumption ratio:  $V_t \equiv \frac{y_t}{M_{t+1}/P_t} = \left(\frac{P_t c_t}{M_{t+1}}\right) \frac{y_t}{c_t}$ . To the extent that income rises temporarily from a goods productivity shock,  $y_t/c_t$  will increase, increasing velocity as in Cooley and Hansen (1995) and Friedman and Schwartz (1963).<sup>3</sup> With the impact of credit and money shocks on  $\frac{P_t c_t}{M_{t+1}}$ , the temporary income channel can be of relatively less importance.

Shocks to velocity effect the growth rate ( $g_t$ ) through the effect on the percent of labor employed ( $1 - x_t$ ); this can be seen intuitively by deriving the balanced-path growth rate as  $1 + g_t = (\beta[1 + A_H(1 - x_t) - \delta_h])^{1/\theta}$  and the marginal rate of substitution between goods and leisure as  $\frac{x_t}{\Psi c_t} = \frac{1 + a_t R_t + (1 - a_t)\gamma R_t}{w_t h_t}$ . A positive money shock increases  $R_t$  and the goods shadow price  $[1 + a_t R_t + (1 - a_t)\gamma R_t]$  relative to the leisure shadow price  $w_t$ , induces substitution from goods ( $c_t/h_t$ ) towards leisure ( $x_t$ ), and decreases the growth rate; a positive credit shock in reverse decreases the cost of exchange, induces substitution from  $x_t$  towards  $c_t/h_t$ , increases the employment rate ( $1 - x_t$ ) and  $g_t$ .

Shocks to velocity also involve a Tobin effect on input price and quantity ratios (see Gillman and Kejak, 2005). A positive money shock causes more

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<sup>3</sup>Such an effect from  $y_t/c_t$  on velocity is included econometrically for US data in Gillman, Siklos, and Silver (1997).



leisure, an increase in  $w_t/r_t$ , and an increase in the capital to effective labor ratio  $\frac{k_t}{l_t h_t}$ ; since it is also true that  $1 + g_t = [\beta(1 + r_t - \delta_k)]^{1/\theta}$ , the fall in  $r_t$  goes in tandem with the fall in the marginal product of human capital,  $A_H(1 - x_t)$ . A positive credit shock conversely decreases  $w_t/r_t$  and  $\frac{k_t}{l_t h_t}$ , and increases  $g_t$ . A goods productivity shock directly increases  $r_t$  and  $g_t$ .

### 3 Impulse Responses and Simulations

Standard solution techniques can be applied once growing real variables are normalized by the stock of human capital so that all variables in the deterministic version of the model converge to a constant steady state. We define  $\tilde{c} \equiv c/h$ ,  $\tilde{i} \equiv i/h$ ,  $\tilde{k} \equiv k/h$ ,  $\tilde{m} \equiv M/Ph$  and  $\tilde{s} \equiv (\tilde{k}, 1, 1, z, u, v)$ , log-linearize the equilibrium conditions of the transformed model around its deterministic steady state, and use standard numerical solution methods.

The calibration uses standard parameters for the goods production labor share of  $\alpha = 0.6$ , a factor productivity normalized at  $A_G = 1$ , capital depreciation of  $\delta_k = 0.012$  and  $\delta_h = 0.012$ , leisure preference of  $\Psi = 3.2$ , consumption elasticity of  $\theta = 2$ , and time preference of  $\beta = 0.99$ . The human capital sector is labor only, with factor productivity of  $A_H = 0.12$ . Time division at baseline is that leisure's share is 0.70, goods production time 0.16, and human capital investment time 0.14; labor in credit production is 0.0008, or 0.0008/0.3=0.27% of total productive time.

For nominal factors, the consumption velocity of money is set to the 1972-2003 average of the consumption velocity of M1, at 4.5 ( $a = 0.224$ ). Shock characteristics are set to estimated values from the constructed shocks: persistences of  $\varphi_z = 0.86$ ,  $\varphi_u = 0.93$ ,  $\varphi_v = 0.93$ , standard deviations of  $\sigma_{\varepsilon_z} = 2.39$ ,  $\sigma_{\varepsilon_u} = 0.85$ ,  $\sigma_{\varepsilon_v} = 1.9$ , and correlations of  $\text{corr}(\varepsilon_z, \varepsilon_u) = -0.03$ ,  $\text{corr}(\varepsilon_z, \varepsilon_v) = -0.24$ ,  $\text{corr}(\varepsilon_u, \varepsilon_v) = 0.85$ . The credit sector productivity parameter is set at  $A_F = 1.86$ , and its Cobb-Douglas parameter  $\gamma$  is calibrated using financial industry data at  $\gamma = 0.13$ . The  $\gamma$  is calibrated by first noting that the Cobb-Douglas function implies a decentralized bank sector profit of  $Rq(1 - \gamma)$ : since  $R$  is the unit credit equilibrium price (equal to the real wage divided by the marginal product of labor in credit production, or the

marginal cost), profit equals  $Rq - wfh$  subject to  $q = A_F(fh)^\gamma d^{1-\gamma}$ ; by the CRS technology property,  $\gamma Rq = wfh$ ; so  $Rq(1 - \gamma)$  is profit returned to the consumer (interest dividend on deposits); and  $\gamma Rq$  is the resource cost of the credit. Per unit of credit this is  $\gamma R$ , so  $\gamma$  is the per unit cost of credit divided by  $R$ . Now, since credit is given by  $q = c - m$ , and  $m = ac$ , then  $q = c(1 - a)$  (equation 5). With the calibration of  $a = 0.224$  then  $q = c(1 - 0.224) = c(0.776)$ . Then  $\gamma = (\text{per unit credit cost})/Rc(0.776)$ . The estimate of 100 is used as the average annual cost over the data period at 2006 prices of an exchange credit card (American Express) and it is assumed to reflect the total interest costs of using the annual exchange credit (not roll-over intertemporal credit) for a single person (other ad-on charges such as penalties are not included). Then  $\gamma = 100/Rc(0.776)$ . Using US annual average data for 1972-2003, with  $c = 15780$  at 2006 prices, being per capita consumption expenditure, and  $R = 0.0627$  the 3-month Treasury Bill interest rate (annual basis), then  $\gamma = 100/[(0.0627)15780(0.776)] \simeq 0.13$ .

Sensitivity to alternative values of  $\gamma$  affect mainly the relative effect of money versus credit shocks on velocity. A larger  $\gamma$  makes the interest elasticity of money demand higher, causes money shocks to affect velocity more, credit shocks to affect velocity less, and thereby increases the importance of the money shock relative to the credit shock. Our low calibrated value of  $\gamma$  thus could be viewed as on the conservative side of the importance of money shocks. And note that a value of  $\gamma$  greater than 0.5 is less plausible as this gives a concave marginal cost curve per unit of credit produced, rather than a convex marginal cost that applies for  $\gamma < 0.5$  (Gillman and Kejak, 2005).

The impulse responses (Figure 1) show the effects of the shocks over time, and illustrate the discussion of the effects of shocks on the equilibrium in Section 2. A positive money shock (M) increases velocity (vel), causes an output growth rate (gY) decrease that persists for more than 50 periods, and an increase in the investment to output ratio, as in a positive Tobin effect. Opposite effects occur for a positive credit shock (CR) on the growth rate and investment ratio, with a positive effect on velocity. The productivity shock (PR) increases velocity, the output growth rate, and the investment ratio over time before the effect turns slightly negative and dies out.

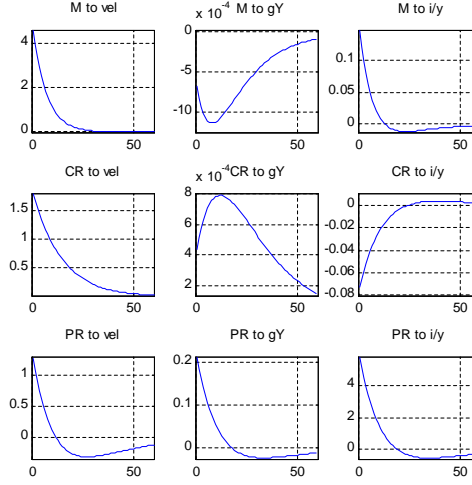


Figure 1: Impulse Responses: Velocity, Output Growth, Investment Ratio

Simulations show that the relative volatility of the output velocity of money, of 1.40, is 75% of the actual 1972-2003 average for the output velocity of M1, of 1.88; this 75% substantially improves on previous work, such as less than 50% in Benk, Gillman, and Kejak (2005), and 57% for the comparable case (of a relative risk aversion coefficient of 2 in Table 3) in Wang and Shi (2006). The model's contemporaneous correlation of velocity with the output ratio  $y/h$  is 0.07, lower than the comparable 0.24 found in the data (where data for  $h$  is described in the Appendix), rather than too high as in Cooley and Hansen (0.95 compared to 0.37 in their data sample). Also, Freeman and Kydland's (2000) simulation shows a real M1 correlation with real output of 0.98 compared to 0.26 in their 1979-1995 subsample. We have a 0.53 output correlation of  $m/h$  compared to the data's  $(M1/P)/h$  output correlation of 0.31 for the 1972-2003 sample; plus, a 1.67 relative volatility of  $m/h$  versus 2.14 in data; a 0.85 correlation of  $c/h$  with output versus 0.79 in data; and a 0.59 relative volatility of  $c/h$  versus 1.03 in data. With only the goods productivity shock active, the  $c/h$  relative volatility is the same, but the velocity relative volatility drops by more than half to 0.56 and  $m/h$  volatility drops in half to 0.83; the model's ability to come close to the data for velocity and  $m/h$  depends on the money and credit shocks being operative.

Shock ordering			Endogenous model			Exogenous model		
CR	PR	M	79%	18%	3%	84%	16%	0%
CR	M	PR	84%	8%	8%	88%	5%	7%
PR	CR	M	5%	92%	3%	5%	95%	0%
M	CR	PR	84%	8%	8%	2%	88%	10%
M	PR	CR	84%	11%	5%	2%	16%	82%
PR	M	CR	5%	89%	6%	5%	14%	81%
<b>Average</b>			<b>PR</b>	<b>M</b>	<b>CR</b>	<b>PR</b>	<b>M</b>	<b>CR</b>
<b>1972-2003</b>			9%	45%	46%	10%	4%	86%
1972-1982			30%	50%	20%	29%	11%	60%
1983-1996			4%	48%	48%	7%	10%	83%
1997-2003			32%	31%	37%	33%	8%	59%

Table 1: Velocity Variance Decomposition, with Different Shock Orderings

## 4 Variance Decomposition Of Velocity

From the shock construction (see Appendix), a standard variance decomposition of velocity is conducted, similar to the variance decomposition for output described in Benk et al (2005) for an exogenous growth case. The endogenous and exogenous growth results are compared in Table 1, for the baseline (five-variable) case of the shock construction, with six possible orderings of the shocks, and for US quarterly data from 1972-2003; here the exogenous growth case used for comparison is the economy set out in Benk et al (2005). For the whole period, the table shows an average effect of 4% for the money shock in exogenous growth but 45% for the endogenous growth model. The credit shock effect on velocity drops from 86% for the exogenous growth results to 46% in endogenous growth. The productivity shock explains an average of 9% of the variance in endogenous growth.

The table also breaks the period into subperiods of 1972-1982, 1983-1996, and 1997-2003. The first subperiod is when the high accelerating inflation rate took place, and credit was restrained by financial sector regulations. The money shock shows a 50% average share, more than twice that of the 20% for credit, while the productivity share is at 30%. In the next subperiod, when financial deregulation was taking place and the inflation rate was much lower but still variable, credit shocks had their highest effect at 48%; money shocks also had a 48% share. In the last subperiod, with a lower, more stable, inflation rate and a significantly deregulated financial market, the money and credit shocks had lower effects, and the goods shock a high of 32%.

The variance decompositions vary with the definition of the subperiod. For example, if the period of 1983-2003 is considered without further subperiods, the goods productivity share is 6% while money and credit shares are 47% and 47% respectively. This masks the fact that the goods productivity played a much bigger role in the latter part of the subperiod, with a share of 32% from 1997-2003, compared to 4% during 1983-1996.

What emerges is that the productivity shock, and the permanent income theory of velocity, takes on more importance during the latter subperiod when there are less episodes of large credit and money shocks. Money shocks are relatively important during the inflation acceleration and deceleration of the 1970s and 1980s; credit is relatively important during financial deregulation.

## 5 Discussion

Prescott (1987) presents a goods continuum with an exogenous division between cash and credit that Freeman and Kydland (2000) and Gillman (1993) make endogenous, resulting in an endogenous velocity. These models involve general transaction costs and a goods continuum that can be cumbersome relative to a more standard single-good model. Alternatively, the Section 2 model has a single good with a credit industry production function from banking microfoundations, allowing plausible credit shocks to sectoral productivity to be identified. This uses the producer side of banking rather than the consumer-side shopping time or trips-to-the bank: consider that with internet banking, shifting funds from savings to current accounts is nearly costless to consumers, getting hold of cash is simple with ubiquitous cash machines or with debit cards at point of purchase, and trips to the bank are optional. However, costs on the production side are real and measurable.

Hodrick, Kocherlakota, and Lucas (1991) use the cash-good, credit good, economy and find that velocity variability, coming from substitution between cash and credit goods, and from the precautionary demand for money when the exchange constraint is not binding, is not fit well relative to evidence for reasonable parameter values. In our model, the exchange constraint always binds, the shocks drive velocity variability, the velocity volatility is within

75% of actual, while the average velocity is matched exactly and parameter specifications are standard except for the credit sector. However a fitness-of-model comparison using the Hodrick et al. approach is not conducted and would be useful.<sup>4</sup>

Ireland (1996) specifies exogenous velocity shocks and productivity shocks, and shows how to maintain the Friedman optimum in the face of such shocks using various money supply regimes. In our model, with an endogenous velocity that is affected by various shocks, it would be interesting to derive how the effects on velocity could be offset through money supply rules in order to establish the optimum or, more topically, an inflation target.

## 6 Conclusion

The paper extends a standard monetary real business cycle by setting it within endogenous growth and adding credit sector shocks. A large portion of the variability of velocity found in the data is simulated in the model, an advance for the neoclassical exchange model. While the standard explanation focuses on the goods productivity shock only in explaining velocity in an exchange economy, here two other factors combine together to play an important role. Shocks to the money supply growth rate have a significant impact on velocity, especially during the high inflation period; credit shocks, found to have an important impact on GDP during the deregulatory era, for example in Benk, Gillman, and Kejak (2005), also effect velocity strongly during this period. Thus while temporary income deviations can be dominant, as in Friedman and Schwartz's (1963) permanent income hypothesis explanation of velocity, during times when money supply growth rates and credit markets are significantly shocked, these other factors can dominate swings in velocity.

The results suggest for example that episodes in monetary regimes could cause different degrees of money supply shocks. This can help explain why there might be higher inflation persistence in the 1970s and 1980s, and less

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<sup>4</sup>See Basu and Dua (1996) for and Hamilton (1989) for other empirical considerations in testing velocity in related cash-good/credit-good models.

such persistence during the inflation targeting period, a possible topic for future work. It might also be a useful extension of this methodology to examine jointly the effects of the shocks on GDP as well as on velocity with a view towards explaining whether having the credit outlet to increase velocity can take pressure off GDP volatility. If so this could be viewed as part of the Jermann and Quadrini (2006) thesis that financial deregulation and increases in finance activity contributed to the post 1983 moderation in GDP, or even to moderations in GDP experienced in the 1930s and 1950s. Another extension could be to examine money and credit shocks in countries outside of the US. Transition countries, with large inflations post-1989 and subsequent banking deregulations, might also reveal significant roles for money and credit influences. Extension of the model to include intertemporal credit that is intermediated through a costly process similar to that of exchange credit would allow for financial shocks that are more of the banking crisis genre.

## A Appendix: Construction of shocks

Based on the solution of the model from section 2, the log-deviations of the model variables be written as linear functions of the state  $\hat{s}_t = (\hat{k}_t, z_t, u_t, v_t)$ . By stacking the equations, the solution can be written in matrix form as  $X_t = A \begin{bmatrix} \hat{k}_t \end{bmatrix} + B \begin{bmatrix} z_t & u_t & v_t \end{bmatrix}'$ , where  $X_t = \begin{bmatrix} \hat{c}_t & \hat{x}_t & \hat{l}_t & \hat{f}_t & \hat{a}_t & \hat{m}_t & \hat{k}_t' \end{bmatrix}'$ . Given the solution for matrices A and B, the series of shocks  $\begin{bmatrix} z_t & u_t & v_t \end{bmatrix}$  are constructed using data on at least three variables in  $X_t$  plus data for  $\hat{k}_t$ , and then backing-out the solution for the shocks in each period. Identification of the three series of shocks requires at least three variables from  $X_t$ . More variables can be used, with the aim of finding robust solutions for the shocks; in this over-identified case a least-square procedure is used. To do this, we use data for the state variable  $\hat{k}_t$ , plus the normalized variables of  $\widehat{c_t/y_t}$ ,  $\widehat{i_t/y_t}$ ,  $\widehat{m_t/y_t}$ ,  $\hat{f}_t$  and  $\widehat{mplb_t}$ , where  $mplb_t$  represents the marginal product of labor in banking from equation (6). Then we let  $XX_t = AA \begin{bmatrix} \hat{k}_t \end{bmatrix} + BB \begin{bmatrix} z_t & u_t & v_t \end{bmatrix}'$ , where  $XX_t = \begin{bmatrix} \widehat{c_t/y_t} & \widehat{i_t/y_t} & \widehat{m_t/y_t} & \hat{f}_t & \widehat{mplb_t} \end{bmatrix}'$  and the rows of the matrices AA and BB result from the linear combinations of the corresponding rows of

matrices  $A$  and  $B$ . Then the baseline estimated three shocks ( $est$ ) are given by least squares as  $est \begin{bmatrix} z_t & u_t & v_t \end{bmatrix}'_t = (BB'BB)^{-1}BB'(XX_t - AA \begin{bmatrix} \hat{k}_t \end{bmatrix})$ .

Here the data series on  $\hat{k}_t$ , where  $\tilde{k}_t = k_t/h_t$ , and  $\hat{k}_t$  is its log deviation, is constructed with the capital accumulation equation and data on investment, giving  $\hat{u}_t$  (with  $\hat{k}_{-1} = 0$ ), and with the human capital series of Jorgenson and Stiroh (2000), extrapolated forward until 2003. We also use data on labor hours  $f_t$  from the Finance, Insurance and Real Estate sector (FIR), and the wage rate in FIR for the marginal product ( $mplb_t$ ); please see the not-for-publication Appendix for further data description and other details.

A crosscheck of the model calibration is to estimate the shock persistence parameters  $\varphi_z$ ,  $\varphi_u$  and  $\varphi_v$  from the constructed shock series. For this reason we estimate a system from equation (1) by the method of seemingly unrelated regressions (SUR). The resulting estimates of the autocorrelation parameters are 0.86 (0.04), 0.93 (0.03) and 0.93 (0.03) respectively (with standard errors in parentheses), which equal the assumed values and thereby show internal consistency of the calibration. From this estimation, the cross-correlations and variances of the error terms are used in the model simulation in Section 3. The corresponding variance-covariance matrix  $\Sigma_t$  for equation (1) contains the following elements:  $var(\epsilon_{zt}) = 5.698$ ,  $var(\epsilon_{ut}) = 0.720$ ,  $var(\epsilon_{vt}) = 3.617$ ; and  $cov(\epsilon_{zt}, \epsilon_{ut}) = -0.056$ ,  $cov(\epsilon_{zt}, \epsilon_{vt}) = -1.106$ ,  $cov(\epsilon_{ut}, \epsilon_{vt}) = 1.376$ .

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